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Folded Yukawa interaction potential model description of heavy ion elastic scattering

V Ramdev Raj

Department of Physics, Osmania University,
Hyderabad-500 007, India

Abstract : The surface interaction potential obtained from the knowledge of the nuclear densities and effective interactions is reviewed The corresponding ion-ion potential and its theoretical estimates with special reference to Folding potentials, proximity potentials are discussed. The comparison of Folded Yukawa interaction potential with other phenomenological potentials in the decription of Heavy ion elastic scattering is presented with reference to the standard woods-saxon form

Keywords : Heavy ion elastic scattering, Yukawa interaction potential

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1. Introduction

The interaction between two nuclei is determined by using singly Yukawa-folded sharp, uniform distributions for the nucleon densities, and effective nucleon-nucleon forces in Yukawa forms. When these are all folded in, one obtains a result in closed from, which, when the centers of the nuclei are not allowed to approach for more than the sum of their radii, is defined to be the surface interaction. Sometime ago Christensen and Winther [1] have proposed an empirical potential to be used for elastic ion-ion collisions. A similar potential could also be derived in the so-called proximity model [2], which is related to the liquid drop model. Both are exponential functions of surface separations of the colliding nuclei, and differ only in the mass dependences of their respective parameters [3]. Broglia and Winther [4] have tried to establish a basis for such potentials through a folding model, but due to the assymmetric representation of the nuclei in the folding, the results could vary up to 50% upon the exchange of the types of the representations.

Several models have been suggested for the calculation of the nuclear interaction potential. For example, Krappe and Nix [5] have proposed a model in which the interaction energy is calculated as the Yukawa interaction between two nuclear distributions with

sharply defined surfaces and uniform interior. The Yukawa interaction is supposed to contain the combined effect of two diffuse matter distributions interacting *via* some short range interaction. This procedure leads to a simple analytic potential.

A different approach is represented by the proximity formula [2] which expresses the force between two gently curved leptodermous surfaces as a product of a geometrical factor proportional to the mean radius of curvature of the gap between the surfaces and a universal function equal to the interaction energy per unit area between two parallel surfaces. This latter approach is very general and has the advantage of being simple to use, once the problem involving the parallel surfaces has been solved.

In the present talk an analytical model is discussed which enables one to gain insight into the accuracy of some of the various approaches, including the two mentioned above. In the model studied, each of the two interacting objects has a diffuse surface which is generated by folding a Yukawa function into a generating sharp-surface distribution. The interaction energy is subsequently obtained on the basis of a two-body Yukawa interaction.

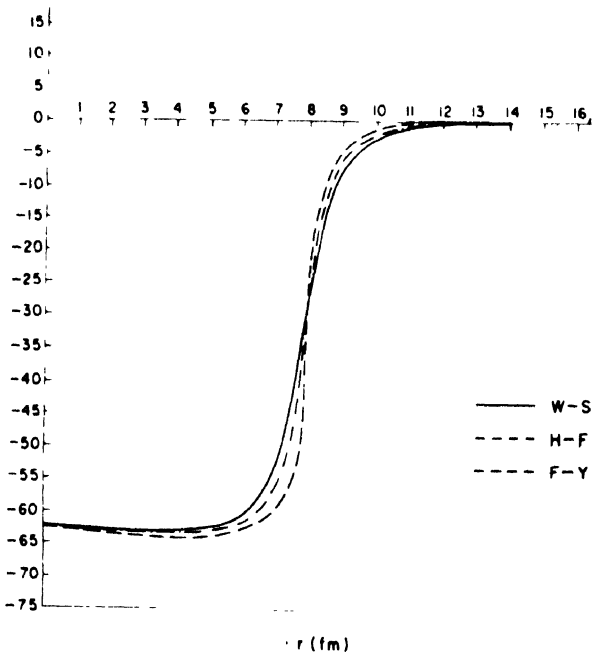

Figure 1. Comparison of the Woods-Saxon, Hill-Ford and Folded-Yukawa potential forms.

This model can be considered a generalization of the Krappe-Nix [5] model. Hence it permits a test of the idea that the interaction can be represented as a single effective Yukawa interaction acting between sharp-surface distributions. Moreover, the model is sufficiently realistic to present a good test case for the proximity formula [2]. So far, such tests have only been carried out for the extreme cases of zero-diffuseness distributions (the Krappe-Nix model) or a zero-range interaction between diffuse surface [2].

2. The folded-Yukawa model

The interaction energy V between two matter distributions ρ_1 and ρ_2 is given by

$$V = -C\iiint \rho_1(\hat{r}_1)\, y_{a_0}(r_{12})\, d^3r_1\, d^3r_2 \tag{1}$$

where the notation

$$y_a = \frac{1}{4\pi a^3}\,\frac{\exp(-r/a)}{r/a} \tag{2}$$

has been introduced. The strength of the interaction is governed by the constant C which is positive for an attractive interaction. The matter density distribution ρ_i ($i = 1, 2$) is obtained by folding a Yukawa function of some range a_i into a generating sharp distribution

$$\rho_i(\hat{r}_i) = \int y_{a_i}(r_{12})\, \hat{\rho}_i(r_2)\, d^3r_2 \tag{3}$$

The starting point for the analytical treatment of this model is the observation that the interaction energy may be calculated as the interaction between the two sharp

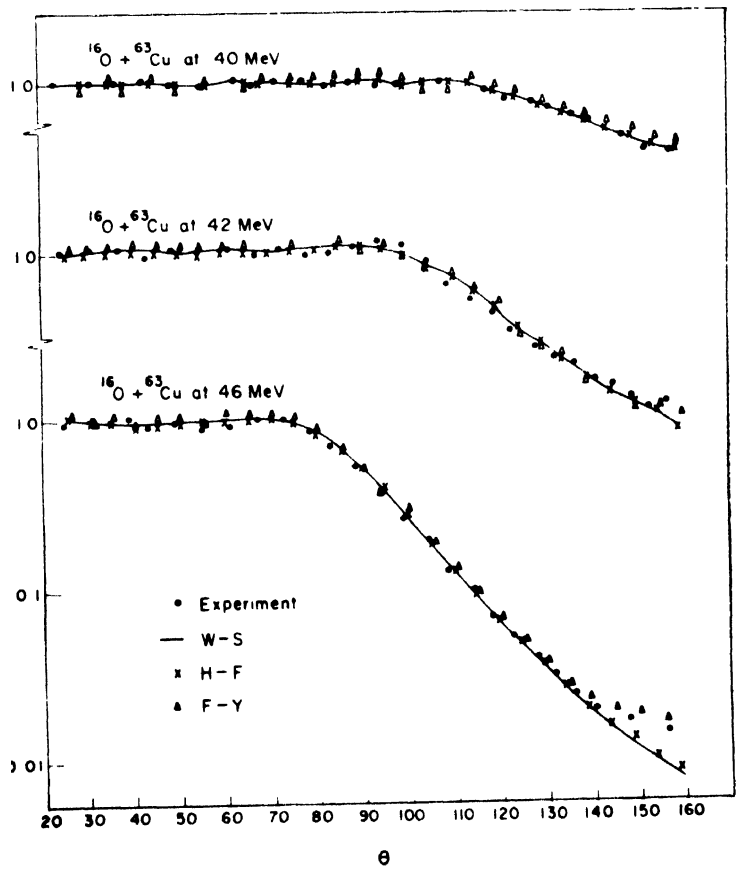

Figure 2. Comparison of the fits to elastic scattering angular distributions for 160 + 63Cu at 42, 44 and 46 MeV(Lab).

generating densties arising from a composite tow-body interaction-cy. This composite interaction is given as the folding product of the three entering Yukawa interactions.

$$y = y_{a_0} * y_{a_1} * y_{a_2} \tag{4}$$

(the symbol * denotes the folding).

The above observation implies that the formulated generalized folded-Yukawa model due to Krappe and Nix [5] in that it calculates the energy by folding some Kernel into generating sharp densities. The generalized model thus applies to all cases covered by the Krappe-Nix model. In particular, the modified surface-energy prescription suggested by Krappe and Nix [5] can be generalized by employing the composite kernal y rather than a single Yukawa function.

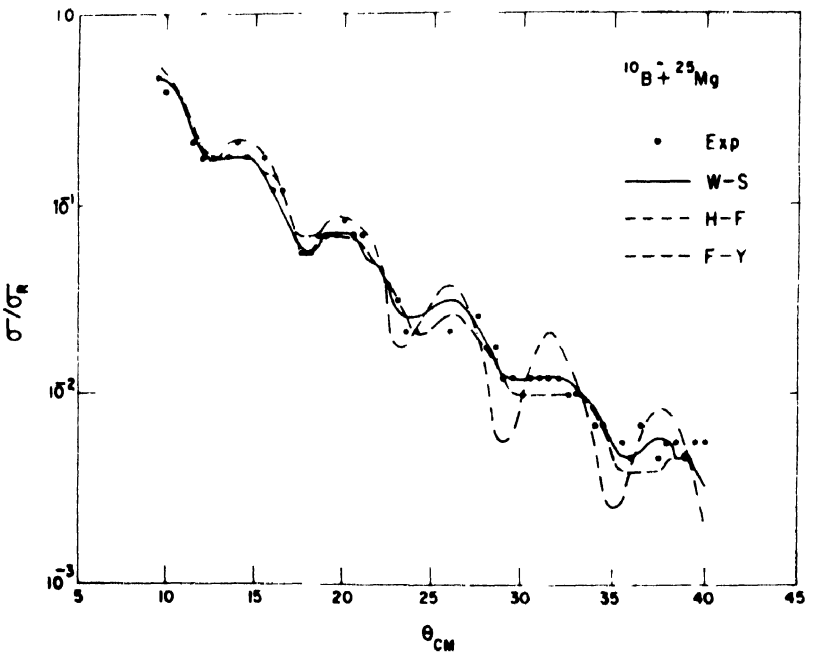

Figure 3. Comparison of the fits to elastic scattering angular distributions for 10B + 25Mg at 87.4 MeV(Lab).

Adopting the above mentioned procedure it is possible to obtain an ion-ion intraction potential which is outlined as follows [7]. The folded-Yukawa parametrized form factor $F(K, r)$ for the charge density $\rho(r)$ as well as the interaction potential $v(r)$ can be given as

$$\rho(r) = \rho_0 F(K, R)$$
$$v(r) = v_0 F(K, R) \tag{5}$$

where ρ_0 is the charge density at the centre and v_0 is the corresponding strength of the potential.

The function $F(K, r)$ is obtained by folding a Yukawa function together with a step function $\theta(R - r')$,

$$F(K,R) = K^2 / 4\pi \int d^3r\, \theta(R-r').\ \frac{\exp(-|r-r'|)}{|r-r'|} \tag{6}$$

The above expression is evaluated by following the procedure outlined above by folding the Yukawa function to a step function defined as

$$(H)\,(R) = \theta(R-r') = \begin{matrix} 1 & \text{for} \quad r < R \\ 0 & \text{for} \quad r > R \end{matrix} \tag{7}$$

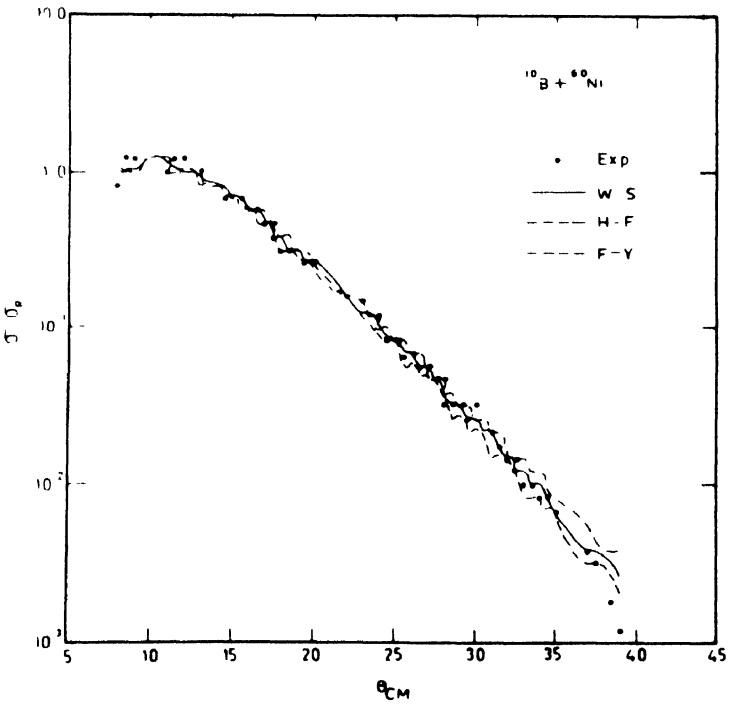

Figure 4. Comparison of the fits to elastic scattering angular distributions for 10B + 60Ni at 87.4 MeV(Lab).

The folding of two Yukawa functions centered at different points is denoted by the symbol (*), *e.g.*,

$$Y(K_1) * Y(K_2) = \int \frac{\exp(-K_1|r_1 - r'|)}{|r_1 - r'|} \cdot \frac{\exp(-K_2|r_2 - r'|)}{|r_2 - r'|} \left(d^3r'\right)$$

$$\frac{4\pi}{K_2^2 - K_1^2} \cdot \frac{\exp\left(-K_1|r_1 - r'|\right) - \exp\left(-K_2|r_2 - r'|\right)}{|r_1 - r_2|} \tag{8}$$

The result of integration is a function of distance [$r_1 - r_2$] only. Equation above can therefore be written as $Y(k_1) * Y(K_2)$

$$= 4\pi/\left(K_1^2 - K_2^2\right)\left[Y(K_1) - Y(K_2)\right]$$

The folding of Y with can also be evaluated explicitly. One finds

$$Y(k) * \Theta(R) = \int \theta(R-r'). \frac{\exp(-K|r-r'|)}{|r-r'|} d^3r'$$

$$= 4\pi / k^2 . F(K, R)$$

with
$$F(K,R) = \begin{cases} 1 - (1+KR)\dfrac{\sin h(kr)}{kr} & r < R \\ R\cos h\, KR - (1/k) \sin h\,(KR)\,(\exp - kr)/r & \text{for } r > R \end{cases} \quad (9)$$

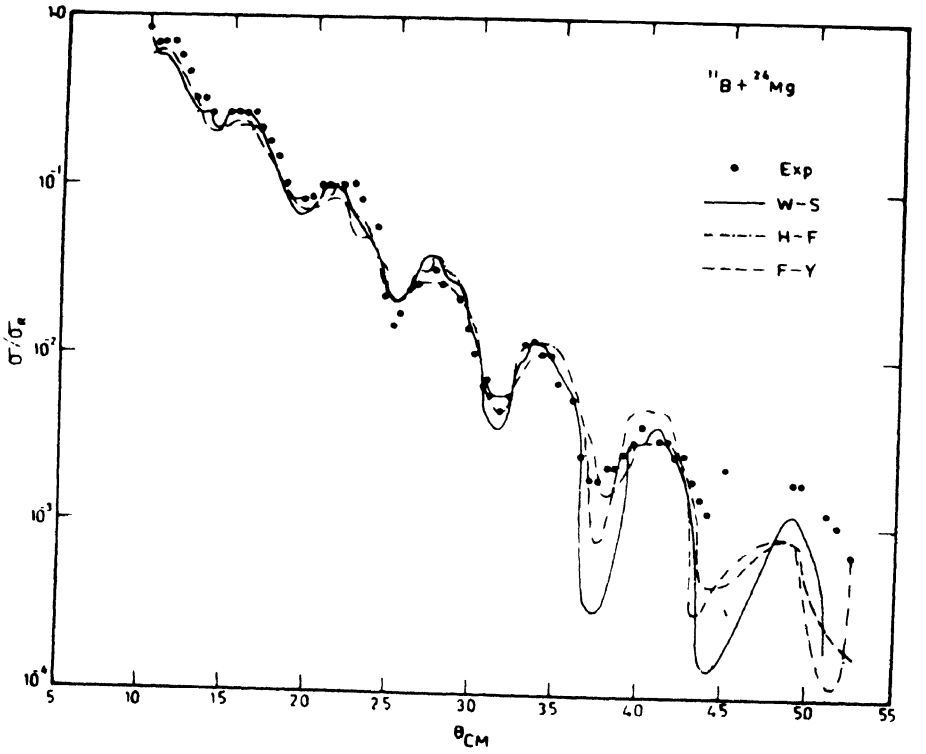

Figure 5. Comparison of the fits to elastic scattering angular distributions for 11B + 24Mg at 79.6 MeV(Lab).

for $KR >> 1$ we find

$$F(K,R) = 1 - 1/2 \exp(-K(R-r)) \quad (10)$$
$$R/2r . \exp(-K(r-R)$$

The function $F(K, R)$ is similar to fermi distribution $[1 + \exp(r - R)/a]^{-1}$ in the tail region. The advantage of this parametrisation is that one may evaluate explicitly multiple folded integrals.

The fermi distribution

$$f(r) = f_0 [1 + \exp(r-c)/a]^{-1} \quad (11)$$

may be approximately written as

$$f(r) = f_0 F(K,R) = f_0 K^2 / 4\pi [Y(K).\ Y(R)] \tag{12}$$

$$f(r) = f_0 F(K,R)$$

$$= f_0 K^2/4\pi\, Y(K) * Y(R) \tag{13}$$

where a fairly good parametrization in the tail region is obtained by Akyuz and Winther [8] for

$$K = 2/3\ (1/a + 1.2/c) \text{ and } R = 0.99c + 0.5a - 0.1 + 0.2/c$$

using these approximations we have analysed the Heavy ion elastic scattering data and found that such approximation was not giving a better fit to the experimental data. We have

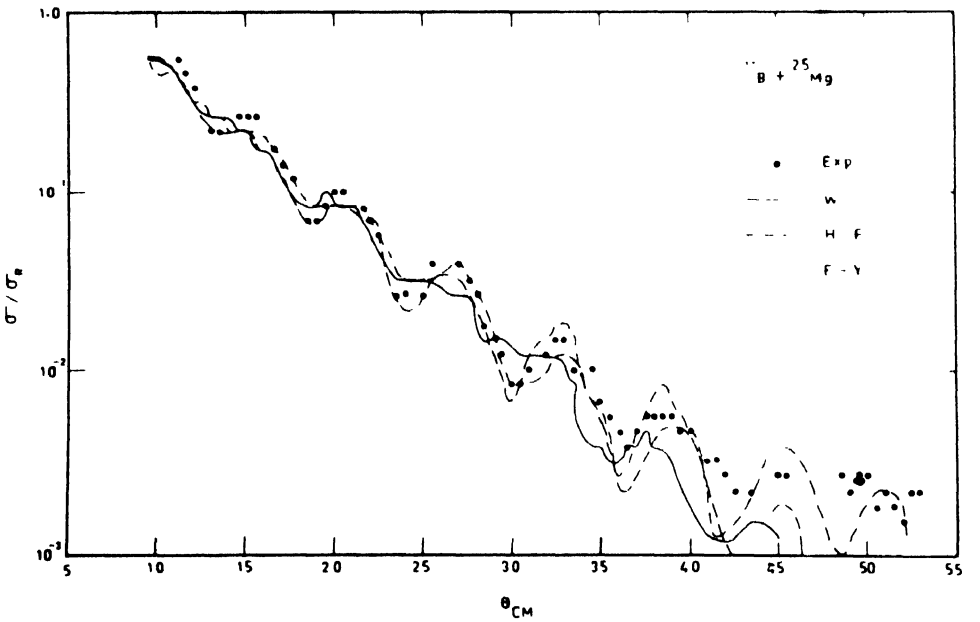

Figure 6. Comparison of the fits to elastic scattering angular distributions for 11B + 25Mg at 79.6 MeV(Lab).

incorporated the procedure of obtaining the better fit to the experimental data by matching the folded Yukawa interaction potential with the woods-saxon potential at the strong absorption radius to obtain the relevant depth and other parameters involved in the analysis. The best fit data so obtained is listed in Table 1.

3. Remarks and conclusions

As expected in the case of single folded potentials the extimation of the real and imaginary strength in folded Yukawa interaction potential is almost double that of Woods-Saxon potential. The analysis is expected to improve by taking density distrubtions obtained for folded Yukawa in the double folded potential. Such analysis was undertaken in the case of

Table 1. 10B + 24Mg Scattering at 87.4 MeV(Lab).

Potential form	V	r_o	a	W	r_u	a_u	x^2
W – S	100.00	1 000	0.674	20 30	1.200	0.892	10.10
F – Y	220 76	1.000	0 735	34 55	1.200	1.059	9.96

10B + 25Mg Scattering at 87.4 MeV(Lab)

Potential form	V	r_o	a	W	r_u	a_u	x^2
W – S	100 00	1.000	0 694	23 70	1.200	0.836	13 42
F – Y	215.96	1.000	0.760	41.40	1.200	0.977	13.86

10B + 60Ni Scattering at 87.4 MeV(Lab)

Potential form	V	r_o	a	W	r_u	a_u	x^2
W – S	40.00	1.014	0 860	70.50	1.014	0 860	1.42
F – Y	79.66	1.014	0.960	141.62	1.014	0.960	2.22

11B + 24Mg Scattering at 79.6 MeV(Lab)

Potential form	V	R_o	a	W	R_u	a_u	x^2
W – S	100.00	0.962	0.721	71.30	0.958	0.814	10.13
F – Y	212 14	0.962	0 797	148.46	0.958	0.918	10.53

11B + 25Mg Scattering at 79.6 MeV(Lab)

Potential form	V	R_o	a	W	R_u	a_u	x^2
W – S	100 00	1 000	0 693	37 00	1.100	0 799	5 52
F – Y	213.45	1 000	0 761	69 89	1 100	0 910	5.54

11B + 27Al Scattering at 79.6 MeV(Lab)

Potential form	V	R_o	a	W	R_u	a_w	x^2
W – S	100.00	1.000	0.685	54.20	1.100	0.692	9.10
F – Y	208 77	1 000	0.754	103 70	1.100	0.772	9.32

11B + 59Co Scattering at 79 6 MeV(Lab)

Potential form	V	R_o	a	W	R_u	a_u	x^2
W – S	40.00	1.135	0.759	44.00	1.135	0.759	1.43
F – Y	63.00	1 135	0.847	69.52	1.135	0.847	2.82

11B + 60Ni Scattering at 79.6 MeV(Lab)

Potential form	V	R_o	a	W	R_u	a_u	x^2
W – S	40.00	0.983	0.896	56.00	0.983	0.896	2.57
F – Y	81.29	0.983	1.005	114.55	0.93	1.005	4.63

Nucleon-Nucleus Scattering with the heavy ion probes. The effects of coupling between channels and the inclusion of quadrupole moment contributions are expected to improve the analysis in inelastic scattering when compared to the conventional Woods-Saxon analysis. The F-Y interaction model predictions will be more useful while dealing with the data where the large angle oscillations are prominent.

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